

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

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Pearson Edexcel International GCSE

Time 2 hours

Paper
reference

4MA1/2HR

Mathematics A

PAPER: 2HR

Higher Tier



You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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P 6 8 7 9 1 A 0 1 2 8



Pearson

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

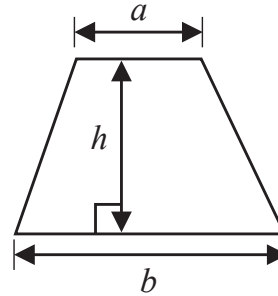
Sum to n terms, $S_n = \frac{n}{2} [2a + (n - 1)d]$

Area of trapezium = $\frac{1}{2}(a + b)h$

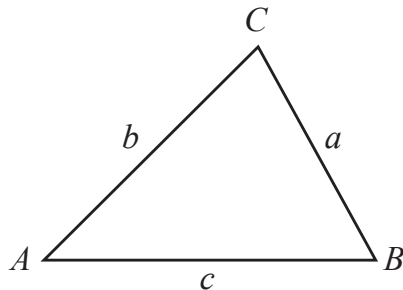
The quadratic equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Trigonometry



In any triangle ABC

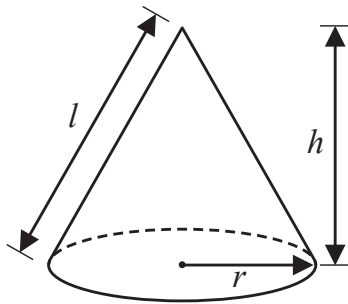
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$

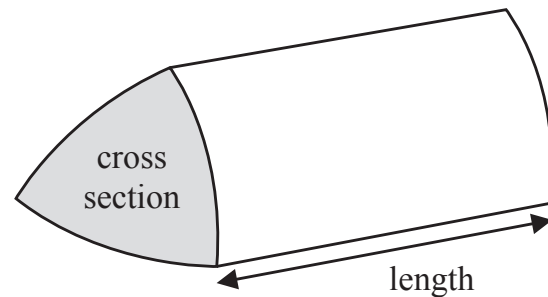
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$



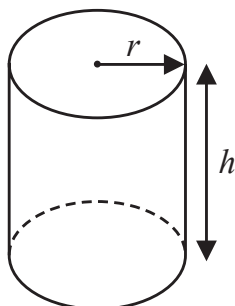
Volume of prism

= area of cross section \times length



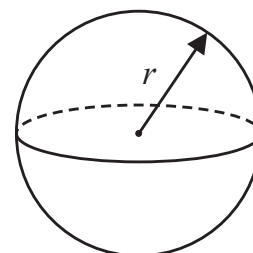
Volume of cylinder = $\pi r^2 h$

Curved surface area of cylinder = $2\pi r h$



Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

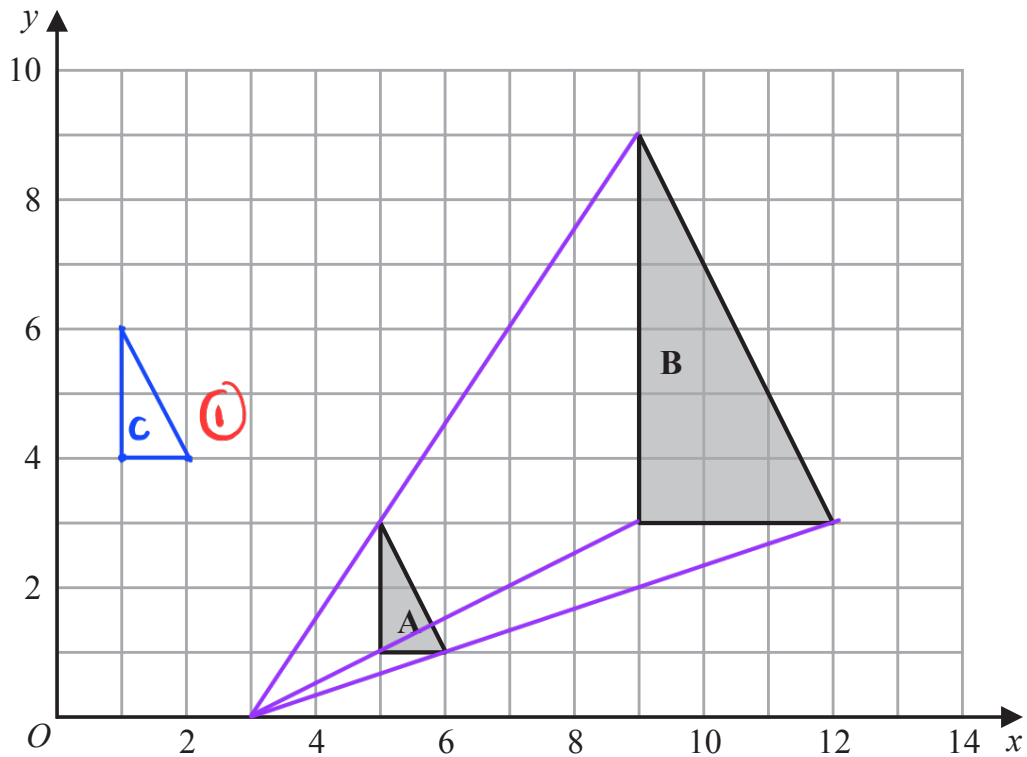


Answer ALL TWENTY FIVE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1



(a) Describe fully the single transformation that maps triangle A onto triangle B

Enlargement of scale factor 3 at centre (3,0)

①

①

①

(3)

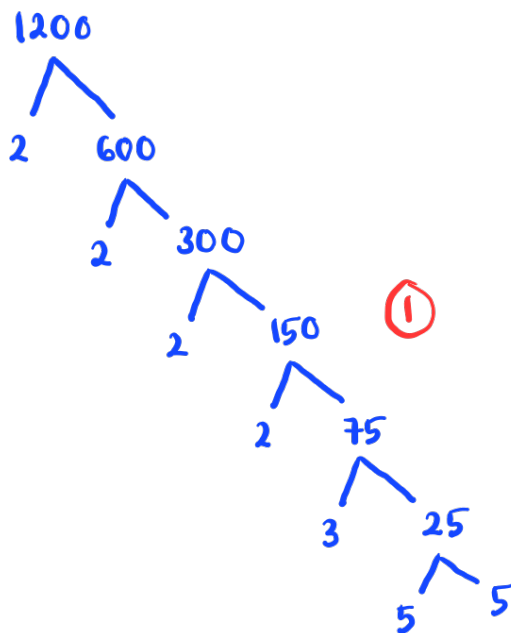
(b) On the grid above, translate triangle A by the vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ - four positions to the left
- three positions upward

Label your triangle C

(1)

(Total for Question 1 is 4 marks)

- 2 Write 1200 as a product of powers of its prime factors.
Show your working clearly.



$$2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 = 2^4 \times 3 \times 5^2$$

① ①

$$2^4 \times 3 \times 5^2$$

(Total for Question 2 is 3 marks)

3 Alberto, Bill, Candela and Diana are four friends.

Here is some information about the height of each of these friends.

Alberto's height is 158 cm.

Bill's height is 175 cm.

Candela's height is greater than Diana's height.

The median height of these four friends is 160 cm.

The range of the heights of these four friends is 21 cm.

Work out Candela's height and Diana's height.

154 158 162 175
y A x B

Since median = 160 cm,

$$\frac{158 + x}{2} = 160$$

$$x = 162 \text{ cm} \quad (1)$$

Since Candela's height is higher than Diana's,

$$x = \text{Candela's height} = 162 \text{ cm}$$

Since range = 21 cm,

$$175 - 21 = y = 154 \text{ cm} = \text{Diana's height} \quad (1)$$

Candela 162 (1) cm
Diana 154 cm

(Total for Question 3 is 3 marks)

4 $\mathcal{E} = \{9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$A = \{\text{multiples of } 3\}$

$B = \{\text{odd numbers}\}$

(a) List the members of the set

(i) $A \cap B$ - is in Set A and Set B

9, 15 (1)

(1)

(ii) $A \cup B$ - is in set A or Set B

9, 11, 12, 13, 15, 17, 18, 19 (1)

(1)

(b) Is it true that $24 \in A$?

Tick one of the boxes below.

Yes

No

Give a reason for your answer.

24 is not between 9 and 20. (1)

\rightarrow is in Set C and not in set B (1)

(1)

Set C has 4 members such that $C \cap B' = \{10, 18\}$

(c) List the members of one possible set C

not in Set B = (10), 12, 14, 16, (18), 20

Members of C: Any 2 numbers except 12, 14, 16, 20

9, 10, 11, 18 (2)

(2)

(Total for Question 4 is 5 marks)

5 The diagram shows a shape made from a square $ABCD$ and 4 identical semicircles.

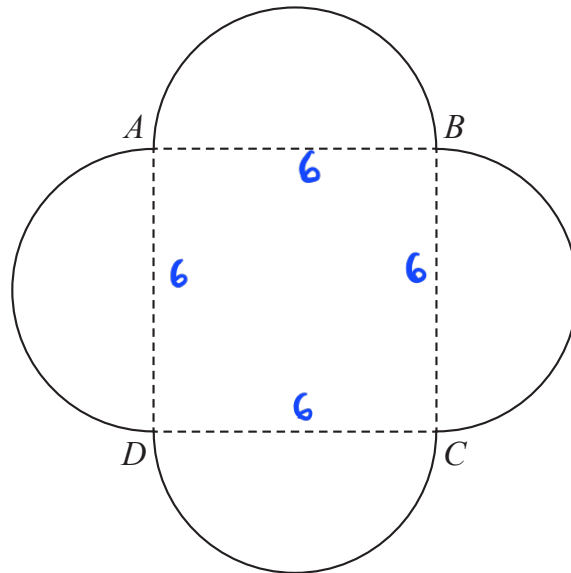


Diagram **NOT** accurately drawn

As shown in the diagram, the semicircles have AB , BC , CD and DA as diameters.

The area of the square is 36 cm^2

Calculate the total area of the shape.

Give your answer correct to one decimal place.

Finding length of sides of $ABCD$:

$$x^2 = 36$$

$$x = 6 \text{ cm} \text{ (1)}$$

\therefore length of side of square = diameter of semicircle = 6 cm

Area of each semicircle:

$$\frac{1}{2} \times \pi \times \left(\frac{6}{2}\right)^2 = \frac{9}{2} \pi \text{ (1)}$$

$$\text{Area of 4 semicircle : } 4 \times \frac{9}{2} \pi$$

$$= 18 \pi$$

Total area: area of square + area of 4 semicircle

$$= 36 + 18 \pi \text{ (1)}$$

$$= 92.5 \text{ cm}^2 \text{ (1)}$$

92.5 cm^2

(Total for Question 5 is 4 marks)

6 (a) Solve $p = \frac{3p - 5}{10}$

Show clear algebraic working.

$$(10)p = 3p - 5 \quad (1)$$

$$10p - 3p = -5 \quad (1)$$

$$7p = -5$$

$$p = \frac{-5}{7} \quad (1)$$

$$p = \frac{-5}{7} \quad (3)$$

(b) Simplify a^0 where $a > 0$

$$1 \quad (1)$$

(c) Simplify fully $\frac{3xy^3}{6x^2y}$

$$\frac{3}{6} \times \frac{x}{x^2} \times \frac{y^3}{y}$$

$$= \frac{1}{2} \times \frac{1}{x} \times y^2$$

$$= \frac{y^2}{2x} \quad (2)$$

$$\frac{y^2}{2x} \quad (2)$$

(d) Factorise fully $10c^3d^2 + 15cd^4$

$$5(2c^3d^2 + 3cd^4)$$

$$= 5c(2c^2d^2 + 3d^4) \quad (1)$$

$$= 5cd^2(2c^2 + 3d^2) \quad (1)$$

$$5cd^2(2c^2 + 3d^2) \quad (2)$$

(Total for Question 6 is 8 marks)

$$7 \quad \frac{2^k}{4^n} = 2^x$$

Find an expression for x in terms of k and n

$$\frac{2^k}{2^{2n}} = 2^x \quad (1)$$

$$2^{k-2n} = 2^x$$

$$x = k - 2n \quad (1)$$

$$x = k - 2n$$

(Total for Question 7 is 2 marks)

- 8 A cinema increased the cost of an adult ticket by 12%
After the increase, the cost of an adult ticket was £18.20
Work out the cost of an adult ticket before the increase.

Ticket cost after increase :

$$100\% + 12\% = 112\% \quad (1)$$

$$\text{Initial ticket cost : } 18.20 \times \frac{100}{112} \quad (1)$$

$$= 16.25 \quad (1)$$

$$\text{£ } 16.25$$

(Total for Question 8 is 3 marks)

- 9 The table gives information about the population, correct to 2 significant figures, of each of five cities in 2018

| City | Population (2018) |
|-----------|-------------------|
| Ahmedabad | 7.7×10^6 |
| Barcelona | 5.5×10^6 |
| Chicago | 8.8×10^6 |
| Lagos | 1.3×10^7 |
| Tokyo | 3.7×10^7 |

- (a) Write 8.8×10^6 as an ordinary number.

8 8 0 0 0 0 0 0

8 8 0 0 0 0 0 0 (1)

(1)

- (b) Which of these cities had the least population in 2018?

Barcelona (1)

(1)

- (c) Work out the difference between the population of Tokyo and the population of Ahmedabad in 2018

Give your answer in standard form correct to 2 significant figures.

$$\text{Tokyo} = 37 \times 10^6$$

$$\text{Ahmedabad} = 7.7 \times 10^6$$

$$\text{Difference} : (37 - 7.7) \times 10^6 \quad (1)$$

$$= 29.3 \times 10^6$$

$$= 2.9 \times 10^7 \quad (1)$$

2.9 $\times 10^7$

(2)

(Total for Question 9 is 4 marks)

10 The diagram shows triangle ABP inside the regular hexagon $ABCDEF$

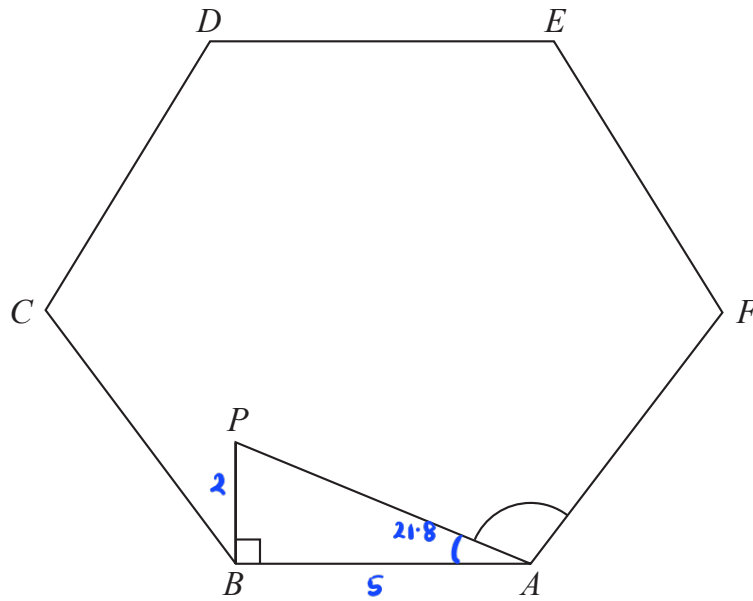


Diagram **NOT** accurately drawn

$$AB = 5 \text{ cm}$$

$$BP = 2 \text{ cm}$$

$$\text{Angle } ABP = 90^\circ$$

Work out the size of angle PAF

Give your answer correct to 3 significant figures.

$$\begin{aligned} \text{Internal angle of hexagon} &= \frac{6-2}{6} \times 180^\circ \\ &= \frac{4}{6} \times 180^\circ \\ &= 120^\circ \quad (1) \end{aligned}$$

$$\tan BAP = \frac{2}{5} \quad (1)$$

$$\begin{aligned} BAP &= \tan^{-1} \frac{2}{5} \quad (1) \\ &= 21.8^\circ \end{aligned}$$

$$\begin{aligned} \text{angle } PAF &= 120^\circ - 21.8^\circ \quad (1) \\ &= 98.2^\circ \quad (1) \end{aligned}$$

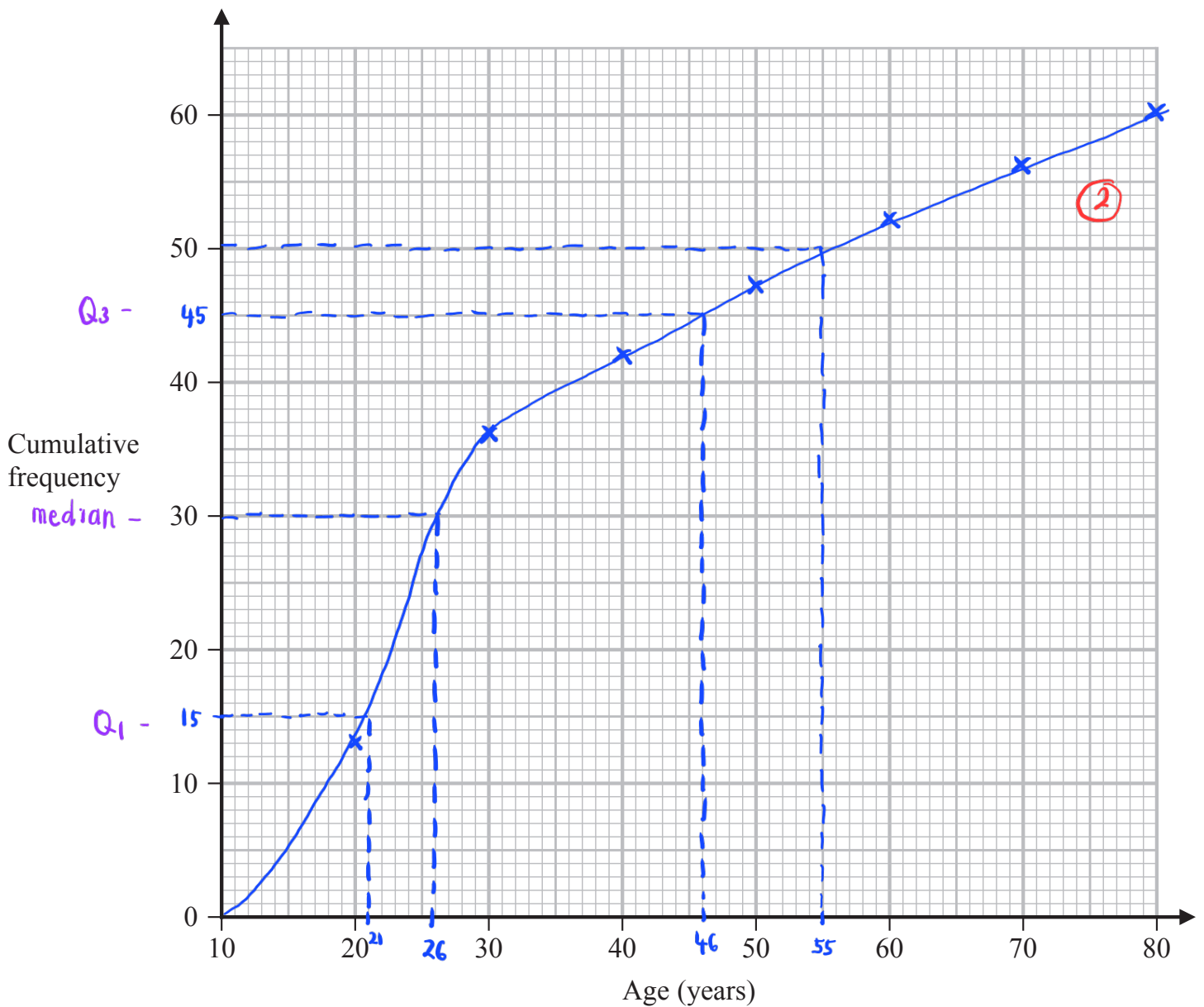
98.2

(Total for Question 10 is 5 marks)

- 11 The cumulative frequency table shows information about the ages of 60 people who went to a gym on Saturday.

| Age (a years) | Cumulative frequency |
|------------------|----------------------|
| $10 < a \leq 20$ | 13 |
| $10 < a \leq 30$ | 36 |
| $10 < a \leq 40$ | 42 |
| $10 < a \leq 50$ | 47 |
| $10 < a \leq 60$ | 52 |
| $10 < a \leq 70$ | 56 |
| $10 < a \leq 80$ | 60 |

- (a) On the grid, draw a cumulative frequency graph for the information in the table.



(2)

Question 11 continued

(b) Use your graph to find an estimate for the median of the ages of these people.

$$\text{Median} = \frac{60}{2} = 30 \text{ (from graph)}$$

..... 26 (1) years
(1)

(c) Use your graph to find an estimate for the interquartile range of the ages of these people.

$$Q_1 = \frac{1}{4} \times 60 = 15 \text{ (from graph)}$$

$$Q_3 = \frac{3}{4} \times 60 = 45 \text{ (from graph)}$$

$$\text{Interquartile range} = Q_3 - Q_1$$

$$= 46 - 21 \text{ (1)}$$

$$= 25 \text{ (1)}$$

..... 25 years
(2)

(d) Use your graph to find an estimate for the number of these people who are older than 55 years.

From graph :

$$60 - 50 \text{ (1)}$$

$$= 10 \text{ (1)}$$

..... 10
(2)

(Total for Question 11 is 7 marks)

12

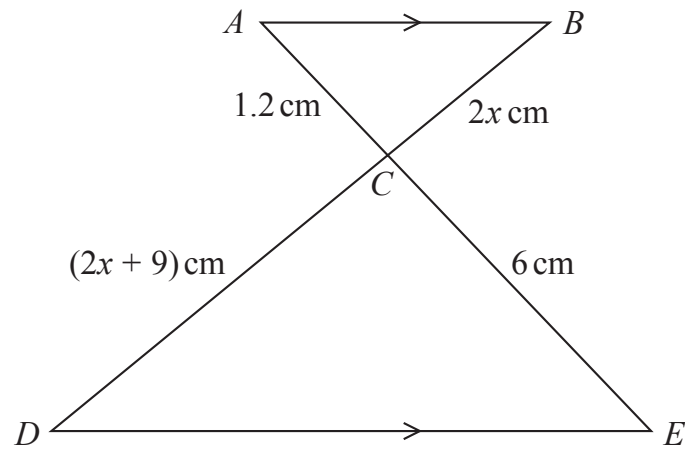


Diagram **NOT**
accurately drawn

ACE and BCD are straight lines.
 AB is parallel to DE

Work out the value of x

Finding scale factor of two triangles :

$$\frac{6 \text{ cm}}{1.2 \text{ cm}} = 5 \quad (1)$$

$$\therefore 5(2x) = 2x + 9 \quad (1)$$

$$10x = 2x + 9$$

$$10 - 2x = 9$$

$$8x = 9$$

$$x = \frac{9}{8} \quad (1)$$

$$x = \frac{9}{8}$$

(Total for Question 12 is 3 marks)

13 The diagram shows a sector AOB of a circle with centre O

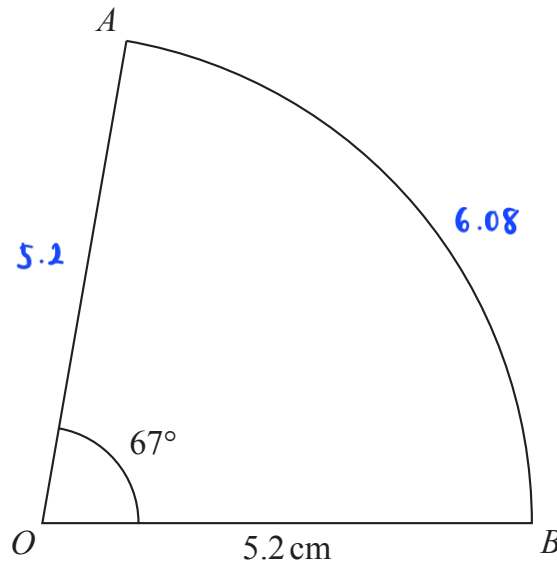


Diagram **NOT** accurately drawn

Angle $AOB = 67^\circ$
 $OA = OB = 5.2$ cm

Calculate the perimeter of the sector.
 Give your answer correct to 3 significant figures.

Circumference of the whole circle :

$$2 \times \pi \times 5.2 = \frac{52}{5} \pi \quad (1)$$

Arc length of the sector AOB :

$$\frac{67}{360} \times \frac{52}{5} \pi = 6.08 \text{ cm} \quad (1)$$

Perimeter of the sector AOB :

$$5.2 + 5.2 + 6.08 = 16.48 \quad (1)$$

$$= 16.5 \text{ (3 s.f.)}$$

..... 16.5 cm

(Total for Question 13 is 3 marks)

14 Ciara throws **four** fair six-sided dice.

The faces of each dice are labelled with the numbers 1, 2, 3, 4, 5, 6

Work out the probability that at least one of the dice lands on an even number.

Probability of a dice to land on even numbers:

$$\frac{3}{6} \equiv \frac{1}{2}$$

Probability of a dice to land on odd numbers:

$$\frac{3}{6} \equiv \frac{1}{2}$$

Tips:

- ① Find probability of all four dices to land on odd numbers.
- ② 1 minus probability in ① to get the probability of at least one dice lands on an even number.

P(all four lands on odd numbers):

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16} \text{ ①}$$

P(at least one of the dice lands on even numbers):

$$1 - \frac{1}{16} = \frac{15}{16} \text{ ①}$$

$$\frac{15}{16}$$

(Total for Question 14 is 3 marks)

15 The diagram shows a kite $ABCD$

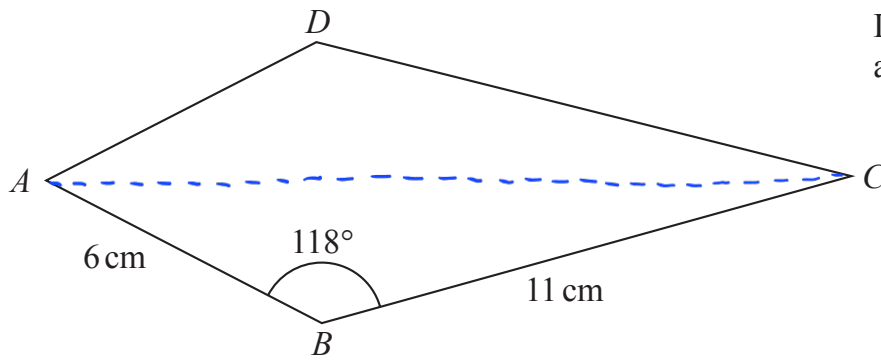


Diagram **NOT** accurately drawn

$$AB = 6 \text{ cm}$$

$$BC = 11 \text{ cm}$$

$$\text{Angle } ABC = 118^\circ$$

Calculate the area of the kite.

Give your answer correct to 3 significant figures.

• Both sides of kite are symmetrical.

Find area of one half of the kite :

$$\begin{aligned} &= \frac{1}{2} \times AB \times BC \times \sin ABC \\ &= \frac{1}{2} \times 6 \times 11 \times \sin 118^\circ \text{ (1)} \\ &= 29.13 \text{ cm}^2 \end{aligned}$$

Area of the whole kite :

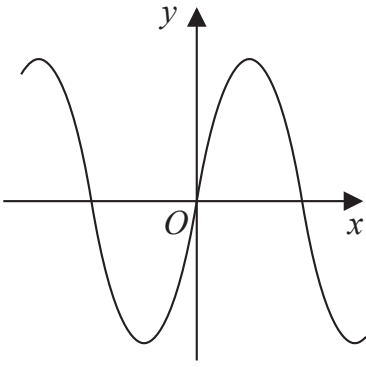
$$\begin{aligned} &2 \times 29.13 \text{ cm}^2 \text{ (1)} \\ &= 58.3 \text{ cm}^2 \text{ (1)} \end{aligned}$$

58.3 cm²

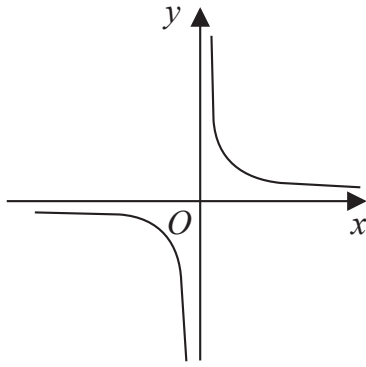
(Total for Question 15 is 3 marks)

16 Here are nine graphs.

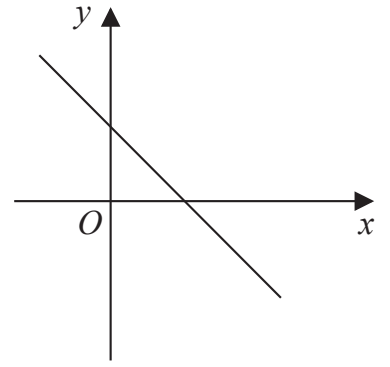
Graph A



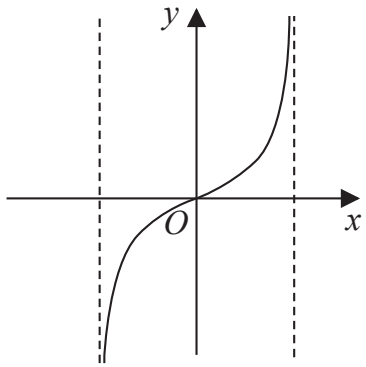
Graph B



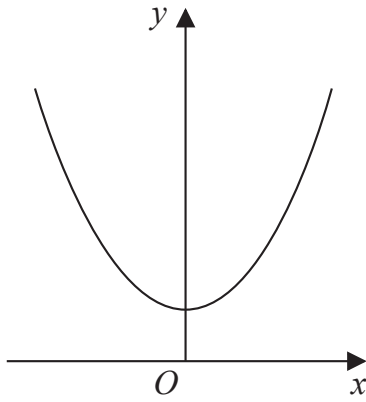
Graph C



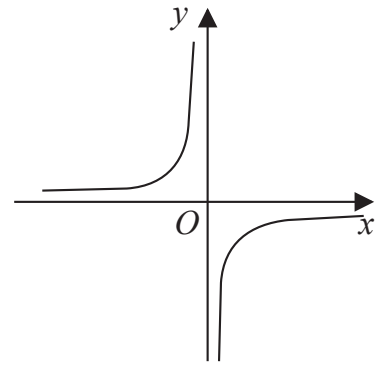
Graph D



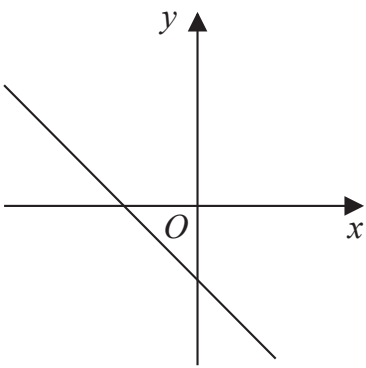
Graph E



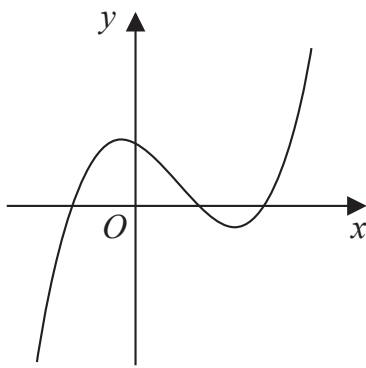
Graph F



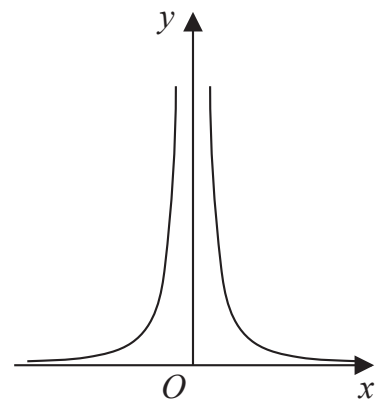
Graph G



Graph H



Graph I



Complete the table below with the letter of the graph that could represent each given equation. Write each answer on the dotted line.

| Equation | Graph |
|-----------------------------|------------|
| $y = -2x + 3$ | C |
| $y = -\frac{1}{x}$ | F |
| $y = \tan x^\circ$ | D |
| $y = (x + 1)(x - 1)(x - 2)$ | H |

y-intercept
at 3

3

(Total for Question 16 is 3 marks)

17 Use algebra to show that $0.\dot{3}4\dot{5} = \frac{19}{55}$

$$\text{Let } x = 0.34545\dots$$

$$100x = 34.545\dots$$

$$100x - x = 34.545\dots - 0.345\dots \text{ (1)}$$

$$99x = 34.2$$

$$x = \frac{34.2}{99} \text{ (2)}$$

$$= \frac{19}{55}$$

(Total for Question 17 is 2 marks)

18 Kaidan and Sonja went on two different car journeys.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

For Kaidan's journey

distance = 80 km correct to the nearest 5 km

time = 2.7 hours correct to 1 decimal place

For Sonja's journey

distance = 33 km correct to 2 significant figures

time = 1 hour correct to the nearest 0.1 hour

Kaidan says,

"My average speed could have been greater than Sonja's average speed."

By considering bounds, show that Kaidan is correct.

Show your working clearly.

Finding upper bound of Kaidan's speed :

$$\text{distance}_{\text{UB}} = 82.5 \text{ km} \quad (1)$$

$$\text{time}_{\text{LB}} = 2.65 \text{ hours}$$

$$\begin{aligned} \text{Speed}_{\text{UB}} &= \frac{82.5 \text{ km}}{2.65 \text{ h}} \\ &= 31.13 \text{ kmh}^{-1} \quad (1) \end{aligned}$$

Finding lower bound of Sonja's speed :

$$\text{distance}_{\text{LB}} = 32.5 \text{ km}$$

$$\text{time}_{\text{UB}} = 1.05 \text{ h}$$

$$\begin{aligned} \text{Speed}_{\text{LB}} &= \frac{32.5 \text{ km}}{1.05 \text{ h}} \\ &= 30.95 \text{ kmh}^{-1} \quad (1) \end{aligned}$$

$$\text{Speed}_{\text{UB}} \text{ of Kaidan} = 31.13 \text{ kmh}^{-1} > \text{speed}_{\text{LB}} \text{ of Sonja} = 30.95 \text{ kmh}^{-1} \quad (1)$$

(shown)

(Total for Question 18 is 4 marks)

19 $f(x) = x^2 - 4$

$g(x) = 2x + 1$

Solve $fg(x) > 0$

Show clear algebraic working.

$$fg(x) = (2x+1)^2 - 4 \quad \textcircled{1}$$

$$fg(x) > 0$$

$$(2x+1)^2 - 4 > 0 \quad \textcircled{1}$$

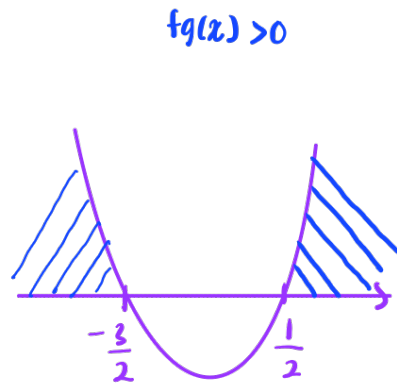
$$(2x+1)^2 > 4$$

$$2x+1 > \pm\sqrt{4}$$

$$2x+1 = 2, \quad 2x+1 = -2$$

$$x = \frac{1}{2}, \quad x = -\frac{3}{2} \quad \textcircled{1}$$

$$x < -\frac{3}{2}, \quad x > \frac{1}{2} \quad \textcircled{1}$$



$$x < -\frac{3}{2}, \quad x > \frac{1}{2}$$

(Total for Question 19 is 4 marks)

20 The centre O of a circle has coordinates $(4, 7)$

The point A , on the circle, has coordinates $(6, 11)$ and AOP is a diameter of the circle.

Find an equation of the tangent to the circle at the point P

Finding coordinates of P :

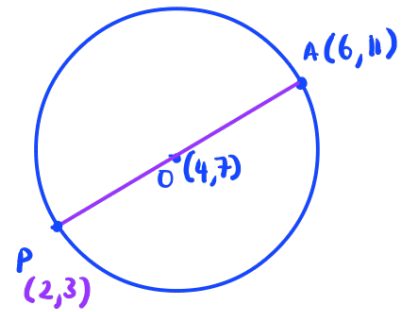
$$x\text{-coordinate} : \frac{6+x}{2} = 4$$

$$x = 2$$

$$y\text{-coordinate} : \frac{11+y}{2} = 7$$

$$y = 3$$

$$P = (2, 3) \quad \textcircled{1}$$



O is the midpoint of PA

Finding gradient of AOP :

$$m = \frac{11-3}{6-2} = \frac{8}{4} = 2 \quad \textcircled{1}$$

Finding gradient of tangent to AOP :

$$m = -\frac{1}{2} = -\frac{1}{2} \quad \textcircled{1}$$

Equation of tangent at P :

$$3 = -\frac{1}{2}(2) + c$$

$$c = 4$$

$$y = -\frac{1}{2}x + 4 \quad \textcircled{1}$$

$$y = -\frac{1}{2}x + 4$$

(Total for Question 20 is 4 marks)

21 Solve the simultaneous equations

$$\begin{aligned}x - 2y &= 3 \\x^2 - y^2 + 2x &= 10 \quad \text{--- ①}\end{aligned}$$

Show clear algebraic working.

$$x = 2y + 3 \quad \text{--- ②}$$

substitute ② into ① :

$$(2y+3)^2 - y^2 + 2(2y+3) = 10 \quad \text{①}$$

$$4y^2 + 12y + 9 - y^2 + 4y + 6 = 10$$

$$3y^2 + 16y + 5 = 0 \quad \text{①}$$

$$(3y+1)(y+5) = 0 \quad \text{①}$$

$$y = -\frac{1}{3}, \quad y = -5$$

substitute y values into ②

$$x = 2\left(-\frac{1}{3}\right) + 3, \quad x = 2(-5) + 3 \quad \text{①}$$

$$x = \frac{7}{3}, \quad x = -7 \quad \text{①}$$

$$x = \frac{7}{3}, y = -\frac{1}{3} \quad \text{and} \quad x = -7, y = -5$$

(Total for Question 21 is 5 marks)

22 The point A with coordinates $(-3, 2)$ lies on the straight line with equation $y = f(x)$

(a) Find the coordinates of the image of the point A on the straight line with equation

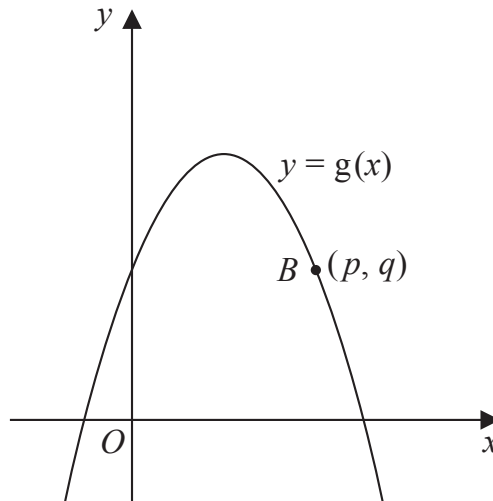
(i) $y = f(x) - 3$ - y is translated 3 positions to the left

$(\dots -3 \dots, \dots -1 \dots)$
(1)

(ii) $y = f\left(\frac{x}{2}\right)$ - x is doubled

$(\dots -6 \dots, \dots 2 \dots)$
(1)

Here is a sketch of part of the curve with equation $y = g(x)$



The point B with coordinates (p, q) lies on the curve.

(b) Find the coordinates of the image of the point B on the curve with equation

$y = -g(x - c)$

where c is a constant.

y is negated x is translated c coordinate to the right

$(\dots p+c \dots, \dots -q \dots)$
(2)

(Total for Question 22 is 4 marks)

23 Express $\left(\frac{20}{x^2-36} - \frac{2}{x-6}\right) \times \frac{1}{4-x}$ as a single fraction in its simplest form.

Simplifying terms in bracket into single fraction:

$$\begin{aligned} & \frac{20}{x^2-36} - \frac{2}{x-6} \frac{(x+6)}{(x+6)} \\ &= \frac{20}{x^2-36} - \frac{2(x+6)}{x^2-36} \quad (1) \\ &= \frac{20 - 2(x+6)}{x^2-36} \\ &= \frac{8-2x}{x^2-36} = \frac{2(4-x)}{x^2-36} \end{aligned}$$

Multiply with the remaining fraction:

$$\begin{aligned} & \frac{2(4-x)}{x^2-36} \times \frac{1}{4-x} \quad (1) \\ &= \frac{2}{x^2-36} \quad (1) \end{aligned}$$

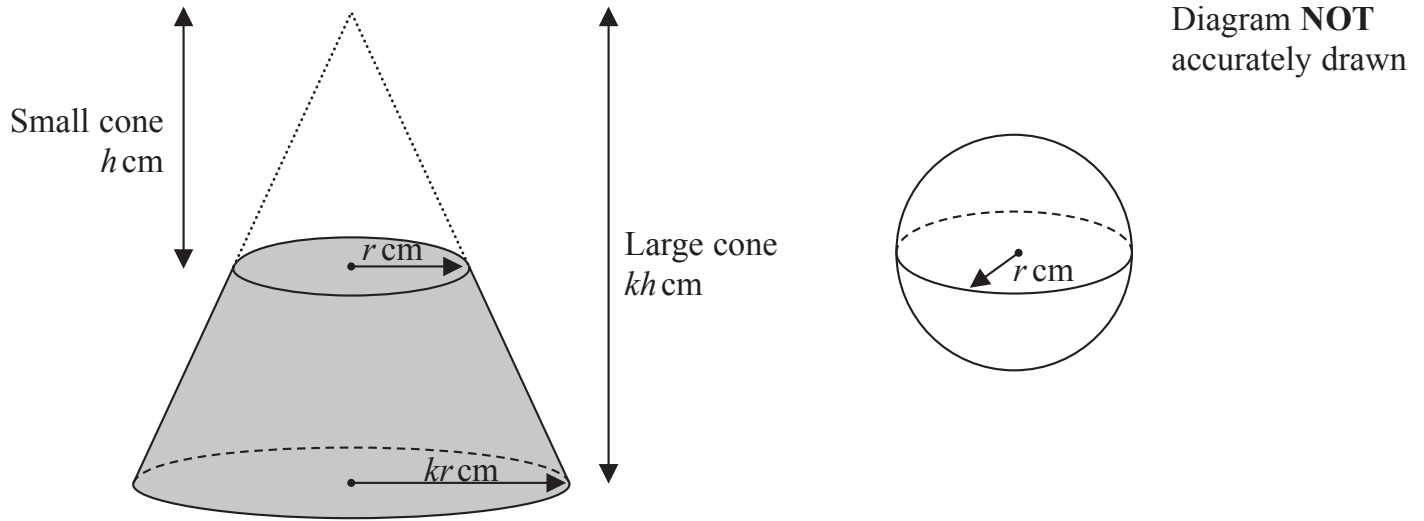
$$\frac{2}{x^2-36}$$

(Total for Question 23 is 3 marks)

24 The diagram shows a frustum of a cone, and a sphere.

The frustum, shown shaded in the diagram, is made by removing the small cone from the large cone.

The small cone and the large cone are similar.



The height of the small cone is h cm and the radius of the base of the small cone is r cm. The height of the large cone is kh cm and the radius of the base of the large cone is kr cm. The radius of the sphere is r cm.

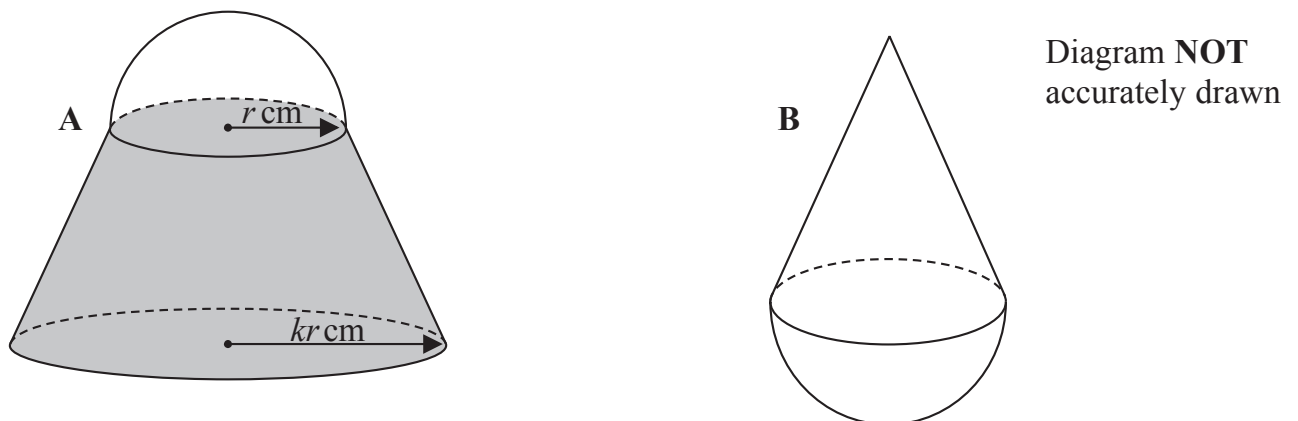
The sphere is divided into two hemispheres, each of radius r cm.

Solid **A** is formed by joining one of the hemispheres to the frustum.

The plane face of the hemisphere coincides with the upper plane face of the frustum, as shown in the diagram below.

Solid **B** is formed by joining the other hemisphere to the small cone that was removed from the large cone.

The plane face of the hemisphere coincides with the plane face of the base of the small cone, as shown in the diagram below.



The volume of solid A is 6 times the volume of solid B.

Given that $k > \sqrt[3]{7}$

find an expression for h in terms of k and r

Volume of each hemisphere :

$$\begin{aligned} \frac{1}{2} \times \text{volume of sphere} &= \frac{1}{2} \times \frac{4}{3} \times \pi \times r^3 \\ &= \frac{2}{3} \pi r^3 \quad \textcircled{1} \end{aligned}$$

Volume of small cone :

$$= \frac{1}{3} \pi r^2 h$$

Volume of frustrum :

Volume of large cone - Volume of small cone :

$$\begin{aligned} \frac{1}{3} \times \pi \times (kr)^2 \times kh - \frac{1}{3} \times \pi \times r^2 \times h \\ = \frac{1}{3} \pi r^2 h (k^3 - 1) \quad \textcircled{1} \end{aligned}$$

Volume of Solid A :

$$= \frac{1}{3} \pi r^2 h (k^3 - 1) + \frac{2}{3} \pi r^3$$

Volume of Solid B :

$$= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 \quad \textcircled{1}$$

$$\therefore \frac{1}{3} \pi r^2 h (k^3 - 1) + \frac{2}{3} \pi r^3 = 6 \left(\frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 \right) \quad \textcircled{1}$$

$$\frac{1}{3} \pi r^2 h (k^3 - 1) + \frac{2}{3} \pi r^3 = 2\pi r^2 h + 4\pi r^3$$

$$\frac{1}{3} h (k^3 - 1) + \frac{2}{3} r = 2h + 4r$$

$$h(k^3 - 1) + 2r = 6h + 12r$$

$$h(k^3 - 1) - 6h = 10r \quad \textcircled{1}$$

$$hk^3 - 7h = 10r$$

$$h = \frac{10r}{k^3 - 7} \quad \textcircled{1}$$

$$h = \frac{10r}{k^3 - 7}$$

(Total for Question 24 is 6 marks)

25 $ABCD$ is a parallelogram and ADM is a straight line.

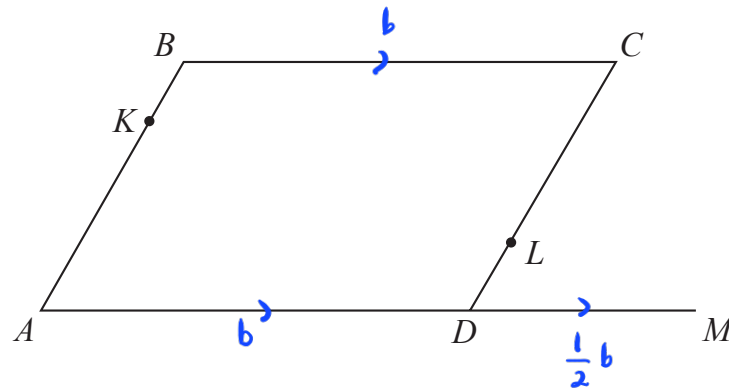


Diagram **NOT** accurately drawn

$$\vec{AB} = \mathbf{a} \quad \vec{BC} = \mathbf{b} \quad \vec{DM} = \frac{1}{2}\mathbf{b}$$

K is the point on AB such that $AK:AB = \lambda:1$
 L is the point on CD such that $CL:CD = \mu:1$
 KLM is a straight line.

Given that $\lambda:\mu = 1:2$

use a vector method to find the value of λ and the value of μ

$$\vec{AK} = \lambda \underline{a}, \quad \vec{KB} = (1-\lambda) \underline{a}, \quad \vec{CL} = -\mu \underline{a}, \quad \vec{DL} = (1-\mu) \underline{a} \quad \textcircled{1}$$

$$\begin{aligned} \vec{KL} &= \vec{KA} + \vec{AD} + \vec{DL} \\ &= -\lambda \underline{a} + \underline{b} + (1-\mu) \underline{a} \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} \vec{LM} &= \vec{LD} + \vec{DM} \\ &= -(1-\mu) \underline{a} + \frac{1}{2} \underline{b} = -(1-2\lambda) \underline{a} + \frac{1}{2} \underline{b} \quad \textcircled{1} \end{aligned}$$

$\mu = 2\lambda$

$$\begin{aligned} \vec{KM} &= \vec{KA} + \vec{AD} + \vec{DM} \\ &= -\lambda \underline{a} + \underline{b} + \frac{1}{2} \underline{b} \end{aligned}$$

$$\vec{LM} = x \vec{KM}$$

$$\underline{a} : 2\lambda - 1 = -\lambda x \quad \textcircled{1}$$

$$\underline{b} : \frac{1}{2} = \frac{3}{2}x$$

$$x = \frac{1}{3} \quad \textcircled{2}$$

Substitute $\textcircled{2}$ into $\textcircled{1}$:

$$2\lambda - 1 = -\frac{1}{3}\lambda, \quad \lambda = \frac{3}{7} \quad \textcircled{1}, \quad \mu = 2 \times \frac{3}{7} = \frac{6}{7} \quad \textcircled{1}$$

$$\lambda = \frac{3}{7}$$

$$\mu = \frac{6}{7}$$

(Total for Question 25 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS